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Attrition Analysis and Non-Homogeneous Customer Populations

By Richard K. Ellsworth

Abstract: Attrition analysis has been accepted by the valuation profession as a simple method to establish customer population life characteristics when informational limitations preclude the use of the retirement rate method. Many customer account populations are assumed to be homogeneous whereby individual accounts possess similar characteristics with regard to size and life characteristics. However, customer populations are frequently composed of large and small accounts with the large accounts representing a significant percentage of revenue. In these instances, a revenue-based attrition analysis should be used with care to estimate the life characteristics and value associated with the customer population.

Attrition analysis is a popular method used to estimate the life characteristics of customer account populations for valuation purposes. Customer account populations are potentially quite diverse with some being quite homogeneous, while others are non-homogeneous. For homogeneous populations, the analytics surrounding the development of customer account life expectancy is straightforward because the customer accounts possess relatively similar characteristics. In contrast, a non-homogeneous population is comprised of large and small customer accounts, with the large accounts representing a significant percentage of revenue when compared with the aggregate population. In these instances, a revenue-based attrition analysis is frequently used to estimate customer population life characteristics.

The recognition and fair value reporting of intangible assets has received particular attention for financial reporting purposes. Customer relationships have been identified as one of the five major categories of intangible assets as part of Financial Accounting Standards Board statements SFAS 141, Business Combinations and SFAS 142, Goodwill and Other Intangibles. Because of the wasting nature of customer relationships, the valuation process relies on the development of life characteristics for the customer account population as an integral part of the analysis. This article discusses the application of attrition analysis as a method to establish life characteristics for non-homogeneous customer account populations.

Customer-Relationship Intangible Assets

Business enterprises expend significant corporate resources to cultivate and maintain customer relationships as part of the corporate focus to forge valuable customer relationships. Businesses invest in building customer relationships with the expectation of continued business patronage. Individual business enterprises seek to develop a myriad of customer relationships during the course of their corporate existence. Valuable customer relationships develop from systematic and measurable business contact between the customer and the business enterprise, leading to formal and informal relationships that evolve over extended time periods.

Many customer populations are viewed as relatively homogeneous groupings with similar characteristics with regard to revenue per account and life expectancy. The financial and human capital resources devoted to

relationship development represents a valuable corporate investment to predispose customers toward continued patronage with the existing business enterprise. The expectation of ongoing business relationships with the customer population is recognized as a valuable intangible asset. However, irrespective of the strength or longevity of the business relationship, customers choose to terminate their relationship as a consequence of factors such as competitive forces, geographic relocations, and shifting consumer preferences.

Customer Population Characteristics

The value associated with the customer relationship is developed through an understanding of the characteristics associated with the underlying population, including its life characteristics. Newspaper and magazine subscribers, along with cable television subscribers, are examples of populations that are considered to be relatively homogeneous when comparing their revenue per subscriber characteristics. However, customer populations are frequently comprised of accounts that lack homogeneous characteristics, instead exhibiting size variations in revenue per account across the population.

Exponential Survival Distribution

Continuous probability distributions, such as the exponential distribution, are popular mathematical models for the description of population life characteristics. Most commonly associated with the insurance industry — where actuaries study human birth and death statistics with the intent of establishing life expectancy for the purpose of calculating insurance premiums — the study of population life characteristics is also applied to customer populations where the population declines over time as customers relocate, suffer mortality, or transfer to competing businesses. Customer account populations experience mortality dispersion analogous to that of human life expectancy with variations across industries and business entities.

The exponential distribution is a continuous probability distribution model that is used to describe customer population life characteristics. The exponential distribution contemplates a constant decay pattern over time for the survival function and implies a constant attrition rate across the age cohorts of the population. The constant attrition rate means that the propensity toward retirement of a customer from the population is independent of age. The exponential distribution is described by the following mathematical function:

$$S(t) = e^{-(\frac{t}{c})} \text{ with } t > 0$$

Where

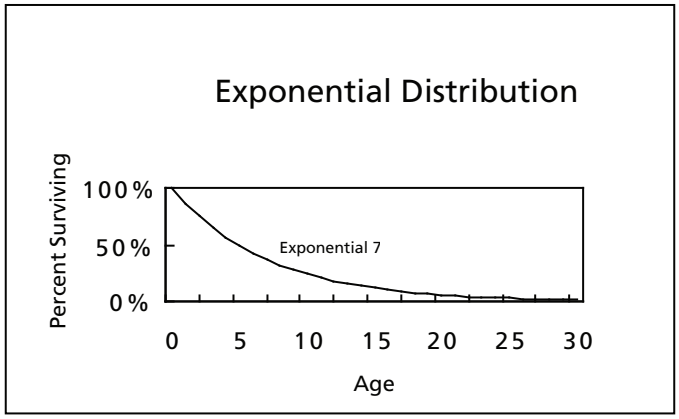
S(t) = Survival percentage at age t

t = Account age

e = Exponential function

c = Scale parameter

Figure 1 graphically illustrates the exponential continuous probability distribution survivor curve profile with 7 as the scale parameter.



A special characteristic of the exponential distribution is that the scale parameter is equal to the average life for the population. The derivation of the average life for a population with an exponential survival curve is presented through the application of integral calculus. The average life for the population is equal to the area under the survivor curve from zero to infinity which is solved mathematically as the integral of the survival curve from age zero to infinity.

$$AL = \int_0^\infty e^{-(\frac{t}{c})} dt$$

$$= -e^{-(\frac{t}{c})} \Big|_0^\infty$$

$$= 0 - (-c)$$

$$= c$$

Where

AL = Average Life

c = scale

Thus, for the exponential distribution, the scale parameter is proven to be equal to the average life for the population.

An additional special feature of the exponential distribution is that the average life and the remaining life expectancy at any age are equal for the population. The equivalency of the average life and the remaining life occurs because of the constant attrition rate or hazard rate with the exponential distribution. The constant attrition rate simplifies the use of the exponential distribution as a continuous probability distribution model descriptor for population life characteristics. This characteristic of the exponential distribution makes attrition analysis readily applicable to establish population life expectancy because knowledge about the

attrition rate permits a direct solution of population life expectancy. The attrition analysis describes the life expectancy for any customer from the population because the exponential distribution life expectancy is independent of age.

Attrition analysis has been shown to be a popular method to establish population life characteristics because of its application simplicity as a method to estimate population life characteristics. Attrition analysis begins with the gathering of information pertaining to account exposures and account retirements for the population from which an attrition rate is then calculated. Life characteristics for the customer account population are then calculated through a consideration of the attrition rate experienced by the population. The observed attrition rate is then used to establish the exponential distribution, which describes the life characteristics for the customer account population.

The complement of the attrition rate is the retention rate, which is an alternative perspective from which to consider account population behavior. For instance, if the attrition rate for a customer population is 10 percent, then the retention rate is one minus the attrition rate or 90 percent. Accordingly, population life expectancy is calculated from the following mathematical relationship:

$$L = -\frac{1}{\ln(R)}$$

Where

L = Life expectancy for the population

ln = Natural logarithm

RR = Retention rate

The population life expectancy calculated from the observed attrition rate is consequently equal to the negative reciprocal of the natural logarithm of retention rate, which is equal to one minus the attrition rate.

While many customer populations are relatively homogeneous and lend themselves to an attrition rate analysis, customer populations can be characterized by a small segment of the population accounting for a dominant share of revenue generation. In these instances, a revenue-based rather than an account-based attrition analysis is frequently performed to establish population life expectancy. The revenue-based attrition analysis implicitly assumes that the life characteristics for the customers are consistent across the population. However, the larger customer accounts tend to experience a lower attrition rate than the smaller customer accounts. The existence of different life characteristics for segments of the population is inconsistent with an underlying assumption for use of the exponential distribution; that is the retirement characteristics for the customer population are constant.

With a population that is comprised of different elements, a more robust analytical approach is necessary for the valuation of the customer population. The solution to this situation is to bifurcate the customer population into two subsets -- large customers and small customers -- and then perform an attrition analysis on each subset. The following example illustrates the concepts surrounding customer population bifurcation for purposes of estimating the value of the customer relationship intangible asset.

Illustrative Example

The attrition rate analysis attributes presented previously are illustrated through an example involving a heterogeneous customer account population. The specific characteristics of the population are that the population is comprised of large accounts that comprise 80 percent of the revenue associated with the company (200 accounts with average revenue of $200,000 per account) while the smaller accounts represent the remaining 20 percent of the revenue (1000 accounts with average revenue of $10,000 per account). Aggregate characteristics of the customer population for the most recent year of financial activity are revenue of $50,000,000 and an observed revenue attrition rate of $6,430,000. An attrition analysis, based on the revenue decline for the total customer population, indicates that the population is experiencing a revenue loss equal to 13.3 percent of the revenue base ($6,643,000 divided by $50,000,000). Life expectancy is then solved as the negative reciprocal of the natural logarithm of the retention rate. The attrition analysis yields a life expectancy of 7 years for the customer account population.

Further assumptions regarding the population profitability to be used to establish the value of the customer accounts are as follows: a revenue growth rate of 2.5 percent per year, cost of goods sold for the population is 65 percent of revenue, general and administrative expense is 10 percent of revenue, selling expense is 5 percent of revenue, contributory asset charges are 4 percent of revenue, the tax rate is 40 percent and the discount rate is 10 percent. The performance of a discounted cash flow analysis to estimate the value of the aggregate customer relationship intangible asset yields a value indication of $28,904,000 (as presented in Appendix A). The development of the discounted cash flow analysis includes the tax benefits from amortization of the intangible asset for 15 years as part of the calculation.

The reliance on the aggregate revenue attrition indication from the customer accounts represents a single snapshot of the attrition activity acting on the customer population. However, many customer account populations are comprised of subsets that possess distinctly different characteristics. The large accounts from the population have average revenue that is 20 times the size of the smaller accounts. In addition to greater revenue per account, the larger customer accounts typically exhibit lower attrition rates relative to the small customer accounts. With a combination of large and small customer accounts and disparate attrition characteristics for each size of account, the customer account

population can be examined through a bifurcated analysis of the large and small account segments of the population.

Further disaggregation of the revenue loss indicates that the larger accounts are experiencing a 9.5 percent attrition rate, while the smaller accounts exhibit a 28.4 percent attrition rate. Consequently, the life expectancy for the large accounts is 10 years and the small accounts 3 years. Extending the growth rate, cost of goods sold, general and administrative expense, selling expense, contributory asset charge, tax rate and discount rate, used for the aggregate population to the individual subsets of the population yields a value for the large customer accounts of $28,862,000 (see Appendix B) and a value for the small customer accounts of $3,036,000 (see Appendix C). As a result of the disaggregation process, the indicated value of the disaggregated customer account population is more than 10 percent higher than the indicated value of the aggregated customer population. In addition, the remaining life for the customer population using the disaggregated analysis is 9 years compared with the 7 years produced with the aggregated analysis.

Conclusion

Attrition analysis has been recognized by the valuation profession as a useful analytical technique that provides a straightforward method to develop customer population life characteristics. Customer population life expectancy development is an integral element of the valuation process since account life characteristics are a principal driver of customer relationship value. Attrition analysis utilizes the exponential distribution as a continuous probability distribution model to describe customer population life characteristics. Attrition analysis works well with homogeneous customer populations but understates the value and life characteristics for heterogeneous customer populations. With heterogeneous populations, revenue-based attrition analysis should be applied with caution because the blind application may produce erroneous results with regard to the life characteristics and value attributable to a customer population.

The views expressed in this article are those of the author and do not necessarily represent the views of Deloitte Financial Advisory Services LLP.

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Appendix A
Discounted Cash Flow Analysis
Combined Customer Accounts
($000's)

Year	Revenue	Cost of Sales	Gen & Admin Expense	Selling Expense	Contributory Asset Charge	Amortization	Earnings Before Taxes	Income Taxes	Net Income	Cash Flow	PV Factor	PV Cash Flow
2008	47,705	31,008	4,771	2,385	1,908	1,927	5,706	2,282	3,424	5,350	0.9535	5,101
2009	42,419	27,572	4,242	2,121	1,697	1,927	4,860	1,944	2,916	4,843	0.8668	4,198
2010	37,691	24,499	3,769	1,885	1,508	1,927	4,104	1,641	2,462	4,389	0.7880	3,459
2011	33,528	21,793	3,353	1,676	1,341	1,927	3,438	1,375	2,063	3,989	0.7164	2,858
2012	29,794	19,366	2,979	1,490	1,192	1,927	2,840	1,136	1,704	3,631	0.6512	2,365
2013	26,480	17,212	2,648	1,324	1,059	1,927	2,310	924	1,386	3,313	0.5920	1,961
2014	23,526	15,292	2,353	1,176	941	1,927	1,837	735	1,102	3,029	0.5382	1,630
2015	20,916	13,595	2,092	1,046	837	1,927	1,420	568	852	2,779	0.4893	1,360
2016	18,577	12,075	1,858	929	743	1,927	1,045	418	627	2,554	0.4448	1,136
2017	16,534	10,747	1,653	827	661	1,927	719	287	431	2,358	0.4044	953
2018	14,706	9,559	1,471	735	588	1,927	426	170	256	2,183	0.3676	802
2019	13,057	8,487	1,306	653	522	1,927	162	65	97	2,024	0.3342	676
2020	11,602	7,541	1,160	580	464	1,927	(71)	(28)	(42)	1,885	0.3038	573
2021	10,303	6,697	1,030	515	412	1,927	(278)	(111)	(167)	1,760	0.2762	486
2022	9,173	5,962	917	459	367	1,927	(459)	(184)	(276)	1,651	0.2511	415
2023	8,165	5,307	817	408	327	0	1,306	523	784	784	0.2283	179
2024	7,228	4,698	723	361	289	0	1,156	463	694	694	0.2075	144
2025	6,434	4,182	643	322	257	0	1,029	412	618	618	0.1886	117
2026	5,728	3,723	573	286	229	0	916	367	550	550	0.1715	94
2027	5,052	3,284	505	253	202	0	808	323	485	485	0.1559	76
2028	4,549	2,957	455	227	182	0	728	291	437	437	0.1417	62
2029	4,017	2,611	402	201	161	0	643	257	386	386	0.1288	50
2030	3,603	2,342	360	180	144	0	576	231	346	346	0.1171	41
2031	3,165	2,057	317	158	127	0	506	203	304	304	0.1065	32
2032	2,858	1,858	286	143	114	0	457	183	274	274	0.0968	27
2033	2,534	1,647	253	127	101	0	405	162	243	243	0.0880	21
2034	2,191	1,424	219	110	88	0	351	140	210	210	0.0800	17
2035	1,996	1,297	200	100	80	0	319	128	192	192	0.0727	14
2036	1,791	1,164	179	90	72	0	287	115	172	172	0.0661	11
2037	1,573	1,022	157	79	63	0	252	101	151	151	0.0601	9
2038	1,433	931	143	72	57	0	229	92	138	138	0.0546	8
2039	1,194	776	119	60	48	0	191	76	115	115	0.0497	6
2040	1,129	734	113	56	45	0	181	72	108	108	0.0452	5
2041	965	627	97	48	39	0	154	62	93	93	0.0411	4
2042	890	579	89	45	36	0	142	57	85	85	0.0373	3
2043	811	527	81	41	32	0	130	52	78	78	0.0339	3
2044	727	473	73	36	29	0	116	47	70	70	0.0308	2
2045	639	415	64	32	26	0	102	41	61	61	0.0280	2
2046	546	355	55	27	22	0	87	35	52	52	0.0255	1
2047	448	291	45	22	18	0	72	29	43	43	0.0232	1
2048	459	298	46	23	18	0	73	29	44	44	0.0211	1
2049	353	229	35	18	14	0	56	23	34	34	0.0192	1
2050	361	235	36	18	14	0	58	23	35	35	0.0174	1
2051	247	161	25	12	10	0	40	16	24	24	0.0158	0
2052	253	164	25	13	10	0	40	16	24	24	0.0144	0
2053	259	168	26	13	10	0	41	17	25	25	0.0131	0
2054	266	173	27	13	11	0	43	17	26	26	0.0119	0
2055	136	88	14	7	5	0	22	9	13	13	0.0108	0
2056	140	91	14	7	6	0	22	9	13	13	0.0098	0
2057	143	93	14	7	6	0	23	9	14	14	0.0089	0
						28,904						28,904

Appendix B
Discounted Cash Flow Analysis
Large Customer Accounts
($000's)

Year	Revenue	Cost of Sales	Gen & Admin Expense	Selling Expense	Contributory Asset Charge	Amortization	Earnings Before Taxes	Income Taxes	Net Income	Cash Flow	PV Factor	PV Cash Flow
2008	38,950	25,318	3,895	1,948	1,558	1,924	4,308	1,723	2,585	4,509	0.9535	4,299
2009	36,142	23,492	3,614	1,807	1,446	1,924	3,859	1,543	2,315	4,239	0.8668	3,675
2010	33,599	21,839	3,360	1,680	1,344	1,924	3,452	1,381	2,071	3,995	0.7880	3,148
2011	31,128	20,233	3,113	1,556	1,245	1,924	3,056	1,223	1,834	3,758	0.7164	2,692
2012	28,964	18,827	2,896	1,448	1,159	1,924	2,710	1,084	1,626	3,550	0.6512	2,312
2013	26,673	17,337	2,667	1,334	1,067	1,924	2,344	937	1,406	3,330	0.5920	1,972
2014	24,725	16,071	2,473	1,236	989	1,924	2,032	813	1,219	3,143	0.5382	1,692
2015	22,906	14,889	2,291	1,145	916	1,924	1,741	696	1,045	2,969	0.4893	1,452
2016	21,231	13,800	2,123	1,062	849	1,924	1,473	589	884	2,808	0.4448	1,249
2017	19,713	12,813	1,971	986	789	1,924	1,230	492	738	2,662	0.4044	1,076
2018	18,369	11,940	1,837	918	735	1,924	1,015	406	609	2,533	0.3676	931
2019	16,946	11,015	1,695	847	678	1,924	787	315	472	2,396	0.3342	801
2020	15,715	10,215	1,572	786	629	1,924	590	236	354	2,278	0.3038	692
2021	14,695	9,552	1,470	735	588	1,924	427	171	256	2,180	0.2762	602
2022	13,614	8,849	1,361	681	545	1,924	254	102	152	2,077	0.2511	521
2023	12,470	8,106	1,247	624	499	-	1,995	798	1,197	1,197	0.2283	273
2024	11,564	7,517	1,156	578	463	-	1,850	740	1,110	1,110	0.2075	230
2025	10,918	7,097	1,092	546	437	-	1,747	699	1,048	1,048	0.1886	198
2026	9,912	6,443	991	496	396	-	1,586	634	952	952	0.1715	163
2027	9,176	5,964	918	459	367	-	1,468	587	881	881	0.1559	137
2028	8,734	5,677	873	437	349	-	1,397	559	838	838	0.1417	119
2029	7,919	5,147	792	396	317	-	1,267	507	760	760	0.1288	98
2030	7,411	4,817	741	371	296	-	1,186	474	711	711	0.1171	83
2031	6,873	4,467	687	344	275	-	1,100	440	660	660	0.1065	70
2032	6,303	4,097	630	315	252	-	1,008	403	605	605	0.0968	59
2033	6,081	3,953	608	304	243	-	973	389	584	584	0.0880	51
2034	5,454	3,545	545	273	218	-	873	349	524	524	0.0800	42
2035	5,191	3,374	519	260	208	-	831	332	498	498	0.0727	36
2036	4,911	3,192	491	246	196	-	786	314	471	471	0.0661	31
2037	4,195	2,727	420	210	168	-	671	268	403	403	0.0601	24
2038	3,870	2,516	387	194	155	-	619	248	372	372	0.0546	20
2039	3,967	2,579	397	198	159	-	635	254	381	381	0.0497	19
2040	3,614	2,349	361	181	145	-	578	231	347	347	0.0452	16
2041	3,241	2,107	324	162	130	-	519	207	311	311	0.0411	13
2042	2,848	1,851	285	142	114	-	456	182	273	273	0.0373	10
2043	2,919	1,897	292	146	117	-	467	187	280	280	0.0339	10
2044	2,493	1,620	249	125	100	-	399	160	239	239	0.0308	7
2045	2,556	1,661	256	128	102	-	409	164	245	245	0.0280	7
2046	2,096	1,362	210	105	84	-	335	134	201	201	0.0255	5
2047	2,148	1,396	215	107	86	-	344	137	206	206	0.0232	5
2048	1,651	1,073	165	83	66	-	264	106	158	158	0.0211	3
2049	1,693	1,100	169	85	68	-	271	108	163	163	0.0192	3
2050	1,735	1,128	174	87	69	-	278	111	167	167	0.0174	3
2051	1,778	1,156	178	89	71	-	284	114	171	171	0.0158	3
2052	1,215	790	122	61	49	-	194	78	117	117	0.0144	2
2053	1,246	810	125	62	50	-	199	80	120	120	0.0131	2
2054	1,277	830	128	64	51	-	204	82	123	123	0.0119	1
2055	1,309	851	131	65	52	-	209	84	126	126	0.0108	1
2056	1,341	872	134	67	54	-	215	86	129	129	0.0098	1
2057	687	447	69	34	27	-	110	44	66	66	0.0089	1
						28,862						28,862

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2009	36,142	23,492	3,614	1,807	1,446	1,924	3,859	1,543	2,315	4,239	0.8668	3,675
2010	33,599	21,839	3,360	1,680	1,344	1,924	3,452	1,381	2,071	3,995	0.7880	3,148
2011	31,128	20,233	3,113	1,556	1,245	1,924	3,056	1,223	1,834	3,758	0.7164	2,692
2012	28,964	18,827	2,896	1,448	1,159	1,924	2,710	1,084	1,626	3,550	0.6512	2,312
2013	26,673	17,337	2,667	1,334	1,067	1,924	2,344	937	1,406	3,330	0.5920	1,972
2014	24,725	16,071	2,473	1,236	989	1,924	2,032	813	1,219	3,143	0.5382	1,692
2015	22,906	14,889	2,291	1,145	916	1,924	1,741	696	1,045	2,969	0.4893	1,452
2016	21,231	13,800	2,123	1,062	849	1,924	1,473	589	884	2,808	0.4448	1,249
2017	19,713	12,813	1,971	986	789	1,924	1,230	492	738	2,662	0.4044	1,076
2018	18,369	11,940	1,837	918	735	1,924	1,015	406	609	2,533	0.3676	931
2019	16,946	11,015	1,695	847	678	1,924	787	315	472	2,396	0.3342	801
2020	15,715	10,215	1,572	786	629	1,924	590	236	354	2,278	0.3038	692
2021	14,695	9,552	1,470	735	588	1,924	427	171	256	2,180	0.2762	602
2022	13,614	8,849	1,361	681	545	1,924	254	102	152	2,077	0.2511	521
2023	12,470	8,106	1,247	624	499	-	1,995	798	1,197	1,197	0.2283	273
2024	11,564	7,517	1,156	578	463	-	1,850	740	1,110	1,110	0.2075	230
2025	10,918	7,097	1,092	546	437	-	1,747	699	1,048	1,048	0.1886	198
2026	9,912	6,443	991	496	396	-	1,586	634	952	952	0.1715	163
2027	9,176	5,964	918	459	367	-	1,468	587	881	881	0.1559	137
2028	8,734	5,677	873	437	349	-	1,397	559	838	838	0.1417	119
2029	7,919	5,147	792	396	317	-	1,267	507	760	760	0.1288	98
2030	7,411	4,817	741	371	296	-	1,186	474	711	711	0.1171	83
2031	6,873	4,467	687	344	275	-	1,100	440	660	660	0.1065	70
2032	6,303	4,097	630	315	252	-	1,008	403	605	605	0.0968	59
2033	6,081	3,953	608	304	243	-	973	389	584	584	0.0880	51
2034	5,454	3,545	545	273	218	-	873	349	524	524	0.0800	42
2035	5,191	3,374	519	260	208	-	831	332	498	498	0.0727	36
2036	4,911	3,192	491	246	196	-	786	314	471	471	0.0661	31
2037	4,195	2,727	420	210	168	-	671	268	403	403	0.0601	24
2038	3,870	2,516	387	194	155	-	619	248	372	372	0.0546	20
2039	3,967	2,579	397	198	159	-	635	254	381	381	0.0497	19
2040	3,614	2,349	361	181	145	-	578	231	347	347	0.0452	16
2041	3,241	2,107	324	162	130	-	519	207	311	311	0.0411	13
2042	2,848	1,851	285	142	114	-	456	182	273	273	0.0373	10
2043	2,919	1,897	292	146	117	-	467	187	280	280	0.0339	10
2044	2,493	1,620	249	125	100	-	399	160	239	239	0.0308	7
2045	2,556	1,661	256	128	102	-	409	164	245	245	0.0280	7
2046	2,096	1,362	210	105	84	-	335	134	201	201	0.0255	5
2047	2,148	1,396	215	107	86	-	344	137	206	206	0.0232	5
2048	1,651	1,073	165	83	66	-	264	106	158	158	0.0211	3
2049	1,693	1,100	169	85	68	-	271	108	163	163	0.0192	3
2050	1,735	1,128	174	87	69	-	278	111	167	167	0.0174	3
2051	1,778	1,156	178	89	71	-	284	114	171	171	0.0158	3
2052	1,215	790	122	61	49	-	194	78	117	117	0.0144	2
2053	1,246	810	125	62	50	-	199	80	120	120	0.0131	2
2054	1,277	830	128	64	51	-	204	82	123	123	0.0119	1
2055	1,309	851	131	65	52	-	209	84	126	126	0.0108	1
2056	1,341	872	134	67	54	-	215	86	129	129	0.0098	1
2057	687	447	69	34	27	-	110	44	66	66	0.0089	1
						28,862						28,862

Appendix C
Discounted Cash Flow Analysis
Small Customer Accounts
($000's)

Year	Revenue	Cost of Sales	Gen & Admin Expense	Selling Expense	Contributory Asset Charge	Amortization	Earnings Before Taxes	Income Taxes	Net Income	Cash Flow	PV Factor	PV Cash Flow
2008	8,672	5,637	867	434	347	202	1,185	474	711	913	0.9535	871
2009	6,377	4,145	638	319	255	202	818	327	491	693	0.8668	601
2010	4,684	3,045	468	234	187	202	547	219	328	531	0.7880	418
2011	3,433	2,231	343	172	137	202	347	139	208	411	0.7164	294
2012	2,523	1,640	252	126	101	202	201	81	121	323	0.6512	210
2013	1,856	1,206	186	93	74	202	95	38	57	259	0.5920	153
2014	1,367	889	137	68	55	202	16	7	10	212	0.5382	114
2015	999	649	100	50	40	202	(43)	(17)	(26)	177	0.4893	87
2016	737	479	74	37	29	202	(84)	(34)	(51)	152	0.4448	67
2017	538	350	54	27	22	202	(116)	(47)	(70)	133	0.4044	54
2018	394	256	39	20	16	202	(139)	(56)	(84)	119	0.3676	44
2019	296	192	30	15	12	202	(155)	(62)	(93)	109	0.3342	37
2020	221	144	22	11	9	202	(167)	(67)	(100)	102	0.3038	31
2021	155	101	16	8	6	202	(178)	(71)	(107)	96	0.2762	26
2022	116	75	12	6	5	202	(184)	(74)	(110)	92	0.2511	23
2023	89	58	9	4	4	-	14	6	9	9	0.2283	2
2024	61	40	6	3	2	-	10	4	6	6	0.2075	1
2025	47	31	5	2	2	-	8	3	5	5	0.1886	1
2026	32	21	3	2	1	-	5	2	3	3	0.1715	1
2027	33	21	3	2	1	-	5	2	3	3	0.1559	0
2028	17	11	2	1	1	-	3	1	2	2	0.1417	0
2029	17	11	2	1	1	-	3	1	2	2	0.1288	0
2030	18	12	2	1	1	-	3	1	2	2	0.1171	0
2031	-	-	-	-	-	-	-	-	-	-	0.1065	-
2032	-	-	-	-	-	-	-	-	-	-	0.0968	-
2033	-	-	-	-	-	-	-	-	-	-	0.0880	-
2034	-	-	-	-	-	-	-	-	-	-	0.0800	-
2035	-	-	-	-	-	-	-	-	-	-	0.0727	-
2036	-	-	-	-	-	-	-	-	-	-	0.0661	-
2037	-	-	-	-	-	-	-	-	-	-	0.0601	-
2038	-	-	-	-	-	-	-	-	-	-	0.0546	-
2039	-	-	-	-	-	-	-	-	-	-	0.0497	-
2040	-	-	-	-	-	-	-	-	-	-	0.0452	-
2041	-	-	-	-	-	-	-	-	-	-	0.0411	-
2042	-	-	-	-	-	-	-	-	-	-	0.0373	-
2043	-	-	-	-	-	-	-	-	-	-	0.0339	-
2044	-	-	-	-	-	-	-	-	-	-	0.0308	-
2045	-	-	-	-	-	-	-	-	-	-	0.0280	-
2046	-	-	-	-	-	-	-	-	-	-	0.0255	-
2047	-	-	-	-	-	-	-	-	-	-	0.0232	-
2048	-	-	-	-	-	-	-	-	-	-	0.0211	-
2049	-	-	-	-	-	-	-	-	-	-	0.0192	-
2050	-	-	-	-	-	-	-	-	-	-	0.0174	-
2051	-	-	-	-	-	-	-	-	-	-	0.0158	-
2052	-	-	-	-	-	-	-	-	-	-	0.0144	-
2053	-	-	-	-	-	-	-	-	-	-	0.0131	-
2054	-	-	-	-	-	-	-	-	-	-	0.0119	-
2055	-	-	-	-	-	-	-	-	-	-	0.0108	-
2056	-	-	-	-	-	-	-	-	-	-	0.0098	-
2057	-	-	-	-	-	-	-	-	-	-	0.0089	-
						3,036						3,036

FAS 157: Pathways and Pitfalls in Fair Value of Complex Investments

*By Michael Fussman and Jason Bronstein**

Introduction

FAS 157, conceived in times of easy credit and bubbly stock market, was born at a time of roiled equity and credit markets and a torrent of lawsuits, among them, those alleging mispricing of exotic securities. The usefulness of FAS 157 is likely to be tested in the crucible of adversary proceedings meriting a close look by companies and regulators alike.

General Principles

The Financial Accounting Standards Board issued Statement No. 157, *Fair Value Measurements*, with the stated purpose of improving the consistency and comparability in fair value reporting of nearly all financial assets and liabilities. By creating a single, comprehensive definition of *fair value*, a set of mandatory disclosures, and a recommended framework for valuation, the Board sought to end the growing propensity of companies to employ opaque accounting practices. As private equity firms, hedge funds, and public entities attempt to customize the valuations of their assets and liabilities, they will be forced to standardize the classification of their internal assumptions. More importantly, these firms will be compelled to examine their current fair value strategies, and in the future, develop measures by which to ensure compliance with FAS 157, which is now effective for fiscal years beginning after November 15, 2007.

Although a "standards board" issued FAS 157, the introduction to Appendix A states:

> This Statement sets out a framework for measuring fair value, which refers to certain valuation concepts and practices. However, this statement is not intended to establish valuation standards.

Beyond the compulsory level of certain disclosures, the Statement does not mandate the use of any particular valuation technique or restrict the use of data. (Nor does it govern the accounting for leases [*see* FAS 13] or the accounting for share-based payment [FAS 123R]). The Appendix characterizes 157 more as a general framework than a clearly defined set of procedures; accordingly, it appears more instructive as a means by which to measure transparency than an endorsement for consistency.

Valuing Specialized Investments

Incorporating the new requirements of FAS 157 may not be difficult for simple equity ownership, but for entities significantly invested in illiquid securities, derivatives, and other specialized financial instruments, complying with the new requirements will necessitate special attention. This article discusses a few of these unique, specialized forms of investment that deserve particular attention.

Because illiquid assets are not often priced on public exchanges, they represent a unique challenge to those who value them for financial reporting purposes. For example, assume that a hedge fund buys shares of restricted and non-restricted stock from the same company whose common shares are traded on the NYSE. The restriction prohibits the hedge fund from selling for a year. Assume further that the equity is a start-up company. Clearly, the fund cannot report the fair value of its restricted stock in the same way as its freely traded common stock. Compliance with FAS 157 requires the fund to account for the additional risk associated with the restriction, despite the absence of guidance on how to calculate a discount for the restrictions. FAS 157 suggests that the fund calculate the value of the equivalent number of common of shares of publicly traded common stock, and then apply a discount for the lack of marketability and liquidity.

In this case, we believe that the nature of the restriction as well as the start-up character of the company should determine the size of the discount. The good news is that there are data sources to assist in the analysis. For instance, an FMV Restricted Stock Study (available at BVResources.com) shows that from July 1980 through March 2005, the variation in applied discounts for lack of marketability was greater than 90%, but that the median discount was 14.95%. This demonstrates the importance of applying a discount that accounts for factors such as company size, industry, and annual revenue.

Similarly, closely held securities must also be discounted to reflect a lack of marketability. Such securities are notorious for being difficult to value due to the absence of an efficient market. Moreover, to comply with FAS 157, the marketability discount must be customized to reflect the idiosyncra-

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sies and circumstances attendant to the hypothetical sale of these securities.

PIPEs and Derivatives

PIPEs—private investments in public equity—are another unique equity offering that require the application of customized discounts. Upon issuance, the market price of a PIPE typically decreases in value about 15 percent.[1] Consequently PIPE investors and issuers must value equity at a price that accounts for the imminent decrease in market price after the public announcement of the PIPE's issuance. However, the price decrease due to the "announcement effect" may vary (greatly in some cases), making it essential to consider the facts and circumstances surrounding each PIPE issuance. There exists no standard discount rate for PIPEs.

Derivatives—one of the most specialized and complex forms of investment—are also subject to the guidance of FAS 157. Even though FAS 133's exhaustive provisions contain many rules regarding fair value determination for derivatives, the FASB has sought to provide further transparency by choosing to include derivatives in FAS 157's oversight. Since many derivatives are characterized by modest initial investment and cannot be valued in their entirety using public exchange prices, reporting entities will be compelled to rely almost exclusively on Level 3 inputs[2]. FAS 133 requires disclosure on how a derivative hedges a particular risk, but does not require a listing of assumptions that quantify that risk. Entities that have previously invested in exotic and complex derivatives to hedge risk that is highly idiosyncratic to a specific business (or industry) will no longer enjoy the luxury of quantifying such risk without disclosing support for its calculations. Regardless of a derivative's degree of specialization and complexity, an entity must now provide transparency for every assumption that it uses to quantify that risk.

Derivative risk can be characterized as performance or non-performance related. Performance risk pertains to the possibility of investment loss due to the performance of the underlying asset. Non-performance risk refers to the creditworthiness of the counter-party. Prior to the issuance of FAS 157, non-performance risk was not required to be considered in determining fair value; however, the Rule pays special attention to non-performance risk by recognizing that counter-party default may represent a significant element of risk. An entity must consider its own credit risk as well that of its counter-party; entities must assign a level of non-performance risk to derivatives in the form of liabilities and assets. Most of the time, assuming sound financial health of the parties involved, performance risk will greatly exceed non-performance risk. FAS 133 requires that reporting entities quantify performance risk at the start of the derivative's existence. Later, as the status of the underlying asset's (or liability's) risk is tracked, the reporting entity is responsible for updating fair value to reflect actual performance of the underlying asset (or liability). This is especially true in the case of long term, illiquid derivatives. Reporting entities will be forced to provide transparency as to performance risk.[3]

Investors Taking Initiative

Although it has become commonplace for entities to invest in complex financial instruments, companies in industries that are more likely to hold such investments have taken the initiative to develop their own recommendations and guidelines, incremental to those set out by FAS 157. For example, in the spirit of FAS 157, the Private Equity Valuation Guidelines Group (PEIGG) issued valuation guidelines in March 2007. Expressing the same FAS 157 goals of consistency and comparability, PEIGG set out principles and procedures especially for private equity investment firms. PEIGG suggests that a market approach is most often the optimal valuation premise. For companies that have achieved positive and sustainable operating performance, PEIGG also suggests a performance-multiple methodology.

Notably, PEIGG advises that the discounted cash flow method (DCF) should seldom be used—but we respectfully disagree. Inevitably there will be situations in which adequate guideline companies are unavailable, and in these cases, DCF represents the most accurate and prudent valuation technique. We recognize that consistency may be compromised by relying on a DCF model, but accuracy should be the paramount goal. Similarly, PEIGG recommends that net asset-based (NAV) valuations should be used for businesses that derive their value primarily from underlying tangible assets. Again, we caution that such a recommendation is not always appropriate and typically should be employed only in cases where an asset's cost basis is justifiably lower than an appraised value. In short, we advise that all entities consider the facts and circumstances regarding each individual valuation in selecting the most suitable methodology.

Many features of FAS 157 make the process of fair valuation more arduous than heretofore. One such feature is FAS 157's exclusion of valuations based on the latest round of financing. To mitigate the increased time and effort needed to ensure compliance, PEIGG suggests the creation of a "Valuation Compliance Committee," comprised of the

1 Source: Houlihan Lokey Howard and Zukin; www.hlhz.com/main.asp?p=CORP_PIPEProductOverview.

2 FAS 157 contains a hierarchy of valuation inputs at three distinct levels. Level 1 involves the analysis of observable and publicly available prices that reflect an asset's value. Data observable to the public but that do not, taken alone, indicate the value of an asset, are considered Level 2 inputs. Level 3 inputs require the reporting entity to make certain assumptions in support of its fair value pricing.

3 Although none may equal Enron's and others' level of disregard for performance risk, no doubt new perpetrators will deliberately ignore substantial elements of performance risk to mislead investors. FAS 157 takes a monumental step in exposing such perpetrators.

fund's investor representatives. The Committee should meet periodically to discuss rules for compliance and conduct valuations by estimating a company's value as a whole and then disaggregating such value among its holdings. By establishing such a committee, soliciting its input and implementing its recommendations, a private equity firm may be able to mitigate litigation risks associated with the implementation of FAS 157.

Issues on the Horizon

As FAS 157 is implemented and current investments are revalued, a number of issues appear likely to confront companies and analysts who prepare fair value estimates to comply with the new rules.

1. FAS 157 no longer permits assets to be valued at cost. Investments currently booked at cost—but which have decreased to lower than cost since the original investment was made—must now be shown at the decreased fair value. Conversely, investments whose fair value has increased since the initial investment must now reflect the appreciated value (i.e. the inability to apply a blockage discount to illiquid securities). This increased mandatory disclosure may prompt investors to question not only the accuracy of these adjustments, but also the propriety of an entity's assumption that cost is equivalent to fair value.[4]
2. Balance sheet liabilities may present issues for publicly traded companies, because a liability's value could now be significantly different than before. Reporting entities must account not only for changes in the overall credit markets but also changes in the actual company's credit risk. Any change in the reporting entity's financial situation must be incorporated into the present value calculation as well. For example, assume that a company has a $1000 obligation due in 10 years; current market conditions have moved interest rates up by 1%, and the company's credit rating was downgraded. In calculating a discount rate for the obligation, the reporting entity is required to account for both change in current market conditions and the downgrade in credit rating.
3. Of all the potential reporting difficulties created by FAS 157, transparency is the most delicate. No longer will investors be forced to wonder the how, where, and why of the valuation process; FAS 157 requires that firms lay out a roadmap for them to follow. This is particularly true for the Level 3 valuations that many of the assets held by private equity and hedge funds will require. As FAS 157 is rolled out, those firms that utilize Level 3 valuations will have to disclose the valuation techniques they used as well as a year-over-year reconciliation of the change in value for each asset.

As investors are able to take this information and reconstruct the valuations, they will be able to make their own judgments on a valuation's validity. This knowledge will allow activists to challenge the reporting entity on its valuations, especially if investment returns on exit are lower than the fair value figure the firm disclosed. One prophylactic would be to accompany disclosures with appropriate disclaimers regarding the judgment calls that were required in the valuation analysis. This will force reporting entities to be especially diligent and their analysts to expend many hours performing these fair valuations, to reassure their investors that the valuations are solid and representative of the true value.

Challenges and Opportunities Ahead

The implementation for FAS 157 holds many challenges for those required to adopt its requirements—and many opportunities for valuation analysts to help entities meet the new requirements. These challenges, current (e.g. the development of valuation techniques and time demands on management) or future (e.g. potential investor dissatisfaction and litigation) demand that the reporting entity ensure that its compliance with FAS 157 is complete and accurate. Attention to the details now by analysts and their company clients will serve to mitigate the problems that are certain to arise as FAS 157 takes hold.

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4 During the remarkable growth and ubiquitous private equity acquisitions over the last couple of years, public companies were acquired at substantial premiums to their pre-transaction market price. In many cases, these premiums did not result from competitive bidding wars but rose from the ease of raising capital to leverage the buyout. During this period, PE firms enjoyed the luxury of reporting the fair value of these purchases at the exact price that they had paid. Since this type of fair value reporting is no longer an option, PE firms now have real incentives to reconsider unwarranted price premiums. Price paid i s no longer a sufficient justification for fair value; rather, an acceptable fair value measurement must reflect the price that other market participants would pay for such a buyout.

Valuation Research Notes

By Lawrence M. Levine

This column provides our readership with an opportunity to read summaries of several of the most up-to-date and innovative valuation research being performed today. Valuation research and empirical studies are being published today in record amounts at many of the world's finest universities as well as by many highly regarded practitioners. The papers we will be covering in this column and future columns have not been disseminated widely but encompass all facets of valuations, including valuation discount analyses, valuation methodologies, nuances of valuing early stage companies through valuations of companies in financial distress, fair value issues, and derivatives. Our intent is to present the substance of the paper, but the *Notes* wording may depart from the authors' original text.

We encourage our readership to forward interesting research and articles to us for future inclusion in this column. Contact the Business Valuation Review office at progers-4bvr@cfail.com; mark your emails to the attention of Larry Levine.

The Moderating Effects of Acquisition Premiums in Private Corporations: An Empirical Investigation of Relative S Corporation and C Corporation Valuations

Author: James A. Digabriele, Montclair State University. Working Paper Series. Last Revised May 29, 2007. (http://ssrn.com/abstract=989759).

Summary: This paper studies whether a premium exists for merger and acquisition transactions of S Corporations over C Corporations during January 2000 through November 2006. The study found that indeed a positive premium exists for S Corporations over C Corporations during this period. The results of the regression analysis employed indicate that the magnitude of the S Corporation premium is particularly dependent on: (1) positively on net sales; (2) the premium is higher in instances in which assets are acquired rather than stock sales; and, (3) the premium is higher for the cases in which firms are bought by private buyers versus public buyers.

The paper provides an extensive survey of the valuation controversy of whether it is appropriate to tax-affect the earnings of S Corporations. It highlights the rationale for imputing entity level income taxes where none are actually incurred (i.e., the likely buyers of the company are C Corporations; there is a risk of revocation of the S election; some form of income taxes will eventually be paid on the S Corporation; and, in instances where the minority interest is expected to have significant, sustainable taxable income without accompanying cash distributions). In contrast, the rationale for ignoring taxes includes exemption from corporate income taxes for S Corporations, avoidance of accumulated earnings and personal holding company taxes and personal service corporation issues.

The paper discusses the valuation issues from several cases including the *Robert Dallas v. Commissioner*, *Delaware Open MRI Radiology Associates v. Howard B. Kessler*, as well as the "trilogy" of S Corporation cases, Gross, Adams and *Heck v. Commissioner.*

The author summarizes two other transactional studies on the relative values of S Corporations versus C Corporations: Mattson and Shannon, and Mattson, Shannon and Upton. Both of these studies concluded that there were no valuation differences for S Corporations and comparable C Corporations. In contrast, one other transactional study, the Erickson-Wong study, did find that when an S Corporation can elect an IRC Section 338(h)(i) election, an S Corporation can command a premium. However, the Erickson-Wong findings have been challenged in several instances.

Study Construction: The primary source of the data for the study was extracted from Pratt's Stats private transaction data base, a commonly used data base by appraisers. The data culled were approximately 4,200 transactions containing sales (deal) price, net sales of selling company, company type (C or S Corporation), buyer type (public or private), and transaction type (asset or stock sale). Logarithmic (natural log) transformations were applied to net sales and purchase price to modulate the wide range of data (net sales in the Pratt's Stats data based ranged from $1,400 to over $400 million while the purchase price ranged from $3,000 to $454.0 million).

A multiple linear regression analysis was conducted using sales (deal) price as the dependent variable, and net sales of the selling company, buyer type, transaction type and company type as the independent variables (the dependent variables were further tested statistically to determine if they had a statistical impact on the other independent variables).

Findings: The statistical analyses resulted in the following regression:

S Corporation Premium = 0.088 - .659 Buyer Type – 0.176 Transaction Type + 0.107 Net Sales

Observations:

1. the S Corporation Premium is lower when the buyer of the firm is Public rather than Private;

2. the S Corporation Premium is lower when the transaction type was Stock rather than when it was an Asset sale;
3. S Corporation Premium has a direct relationship with Net Sales – firms with higher Net Sales tend to also show a higher premium for being an S Corporation.

Example:

If it is assumed that the buyer is Private, the transaction is an Asset sale, and Net Sales are equal to the average in the Pratt Stats data base, the coefficient becomes 0.088. Therefore, for a Private Company undergoing an Asset Sale, the S Corporation premium is 8.8%.

Conclusion: This paper presents a multiple regression analysis to show that, ceteris paribus, there exists a positive premium in the relative value of S Corporations over C Corporations. The premium can increase or decrease (or even become negative) given different buyer types, transaction types and net sales.

The Price of Corporate Liquidity: Acquisition Discounts for Unlisted Targets

Author: Micah S. Officer, University of Southern California. *Journal of Financial Economics*, vol. 83, No. 3, pp. 571-598.

Summary: This paper documents large acquisition discounts for stand along private firms and subsidiaries of other firms ("unlisted targets") relative to acquisition multiples for comparable publicly traded targets. Dr. Officer's conclusion is that the acquisition discounts for unlisted targets represents the price paid by the private owner or public corporations for the liquidity provided by the buyer. The results are consistent with the idea that sale prices for unlisted targets are significantly impacted by the need for, and availability of, liquidity.

Dr. Officer presents an interesting analysis for why obtaining and maintaining liquidity can be expensive for corporations and their shareholders. Clearly, owners of privately held firms pay considerable costs to access public pools of liquidity that enable them to diversify their portfolios (i.e., sell stock publicly). Furthermore, research confirms that publicly traded firms appear to hold larger than expected cash balances. There are several reasons for this finding:

- Information disparities between public firms and the markets result in firms holding larger than expected cash balances.
- Public firms want to insure against unexpected liquidity shortages.
- Agency conflicts between stockholders and managers.

This paper investigates the cost of accessing liquidity by selling unlisted assets. Dr. Officer contributes to the literature by examining the cost (i.e., price discount) of obtaining liquidity by selling a subsidiary or an entire unlisted firm. Given that nearly two-thirds of the acquisitions reported by Securities Data Corporation (SDC) are of unlisted targets, this information is extremely important and academics have prepared little research on the merger and acquisition discounts for unlisted targets.

Study Construction: Most empirical studies of valuation differences between public and private firms compare acquisition multiples for unlisted targets to average trading multiples for all comparable publicly traded firms. In contrast, Dr. Officer compares acquisition multiples for unlisted targets to acquisition multiples for portfolios of comparable industry and size-matched publicly traded targets. For each unlisted target, portfolios are formed of comparable acquisitions of publicly traded targets from the SDC data, where comparable acquisitions are those for which the publicly traded target in the same 2 digit-SIC code as the unlisted target, has a deal value within 20% of the unlisted target and is announced within a 3 year window.

The data is generated from SDC Mergers and Acquisitions data base from 1979 through 2003. It includes successful and unsuccessful bids for at least one-half of the target's equity, deal value in excess of $50 million, and only bids that are all cash, all common stock or a mix. The study omits bids with debt or preferred securities. There were approximately 4,200 divestitures of publicly traded firms and another 420 of stand alone as well as subsidiary unlisted targets in the population (the population consisted of approximately 5,000 observations).

Findings: This paper describes the level and determinants of multiples paid to acquire unlisted targets and how these prices vary with the financial performance of the seller and the characteristics of debt and equity markets around the time of the sale. He finds that:

- Unlisted targets sell at a discount of 15% to 30% relative to comparable sales of public firms.
- Almost 70% of unlisted targets are acquired at lower multiples than that offered to acquire comparable public firms.
- Public firms that sell unlisted subsidiaries exhibit symptoms of being liquidity constrained. For example, sellers, relative to comparable other firms, have lower cash balances, cash flow, net working capital, bond ratings and Altman Z-scores. They also had higher leverage.
- The discount from the average acquisition multiple for a comparable publicly traded firm is strongly related to the parent firm's pre-sale stock return performance; the findings were statistically significant that sellers had negative 12 month abnormal returns leading up to the sale.
- While the subsidiaries are small relative to the selling firms (an average of 4% of parent assets), the proceeds

from the sale are large relative to the parent's pre-sale cash balances (on average 105% of existing cash balances).

- The discount accepted when parent firms sold subsidiaries in industries that are unrelated to the parent firm's principal line of business, were almost at twice the discount during periods of poor performance.
- Sale multiples were significantly lower when corporate loan spreads were high. (This is consistent with the notion that the cost to sellers in obtaining liquidity is high when the cost of capital is high.)
- Information costs are factored into the purchase price. As greater information asymmetries exist when acquiring an unlisted target versus a publicly traded firm, due diligence costs are greater. Dr. Officer suggests that another reason for acquisition discounts is that lower relative sale multiples partially mitigate a buyers risk when they perceive they have less than complete information.

Conclusion: Dr. Officer's findings are that unlisted targets are acquired at a 15% to 30% discount relative to comparable publicly traded targets. He concludes by stating that selling to obtain liquidity carries a substantial cost.

Is Cash Flow King in Valuations?

Authors: Jing Liu, University of California at Los Angeles, Jacob K. Thomas, Yale School of Management, and Doron Nissim, Columbia Business School. *Financial Analysts Journal*, March/April 2007.

Summary: Only recently have researchers begun to evaluate the reliability of different valuation models. We included two such articles in last column on this issue. This article concludes that while the perception is that operating cash flows, rather than accounting earnings, are a more accurate means of valuing equity, recent studies suggest that valuations derived from industry multiples based on reported and forecasted accounting earnings are more accurate than valuations derived from reported and forecasted operating cash flows. This study also finds that valuations based on reported accounting earnings dominates valuations based on dividends. Their main finding is that valuations based on forward earnings multiples are considerably more accurate than valuations based on forecasted cash flow multiples.

The researchers argue that earnings should be a more representative value driver because earnings aggregates value relevant information in both cash flows and accruals. However, they state that practitioners prefer to use cash flow multiples, arguing that accruals involve discretion and are often used to manipulate earnings and that non-cash charges deviate substantially from actual value because they are both estimates and derived from historical costs.

Study Construction: In this study, the researchers compare the valuation performance of earnings multiples versus multiples based on two measures of cash flow – operating cash flows and dividends. Interestingly, the sample is derived from 8 different markets.

The benchmark used for comparing valuation methodologies was the traded stock price. It assumes that the traded stock is priced correctly, and focuses on how close valuations based on industry multiples are to traded prices. The objective is to investigate whether earnings or cash flows best represent a summary measure of value vis-à-vis the stock price.

This study obtains data for:

- Cash flow using both operating cash flows and dividends;
- Earnings using both forecasts and reported earnings;
- Individual industries, versus other studies which have been on a more aggregated basis; and,
- Firms in markets outside of the United States.

Valuations using industry multiples relies on a proportional relation between value (pit) for firm i in period t and a value driver (x_{it}), such as earnings, dividends, or operating cash flows:

$$p_{it} = \beta_t x_{it} + \varepsilon_{it}$$

The researchers tested whether pricing errors (εit) are greater when the value driver is earnings versus whether it is a cash flow measure. To determine pricing errors, the researchers first estimated the industry multiple, βt. The industry multiple was calculated separately for each firm and period, by using prices and value drivers for all remaining firms in the same industry. Then the industry multiple times the firm specific value driver (i.e., earnings or cash flow) is compared to the traded price. The difference between the traded price and the multiples based valuation equals the "valuation error". The valuation error with the lower dispersion is considered to be superior. Excluded from the data set were firms with multiple valuations which were negative.

United States Sample

The data set was a sample of nearly 20,000 U.S. firm year observations during the period 1982 through 1999. Industry multiples for book value, operating cash flow, EBITDA, EPS, revenue and consensus analysts' one and two year out EPS forecasts were obtained.

International Sample

This data set consisted of forecasted and reported data from the IBES International Summary and Actual files. This data provides consensus analyst forecasts and reported numbers for different value drivers, monthly. While IBES collects forecasts from 63 countries, the researchers believed that a much smaller number had statistical data. The countries with sufficient forecasts for operating cash flow are Austra-

lia, France, Hong Kong, Taiwan and the UK. The countries for dividend forecasts are Australia, France, Germany, Hong Kong, Japan, South Africa, and the United Kingdom. The initial sample was nearly 1.6 million observations for almost 26,000 firms between January 1987 and September 2004.

Findings:

United States Sample

The researchers observe the following ranking of multiples in descending order:

- Two year out forward earnings;
- One year out forward earnings;
- Reported earnings dominate all other value drivers in pricing precision; and,
- Sales and operating cash flow valuation multiples were found to be the worst performers.

International Sample

The researcher found that in Australia, France, Hong Kong, Taiwan, United Kingdom, Germany, Japan, South Africa:

- Valuations based off of forecasted earnings per share dominated valuations based off of forecasted operating cash flow.
- Valuations based off of forecasted operating cash flows were superior to valuations based off of actual operating cash flow.
- Valuations based off of reported earnings per share were superior to valuations based off of actual operating cash flow.
- Valuations based off of forecasted earnings per share were superior to valuations based off of forecasted dividends per share.
- Valuations based off of forecasted earnings per share were superior to valuations based off of actual earnings per share.
- Valuations based off of forecasted dividends per share were superior to valuations based off of actual dividends per share.

Conclusion: The researchers dispute the notion that cash flow is king in equity valuation. They conclude that earnings forecasts represent substantially better summary measures of firm value than operating cash flows in all countries examined and this relative superiority is observed in most industries they analyzed. They also found that earnings forecasts are better summary measures of value than dividend forecasts. They also found that moving from reported numbers to forecasts improves performance. They authors conclude by stating that their results suggest that proponents of cash flow multiples consider using earnings multiples.

Accounting Discretion and Purchase Price Allocation After Acquisitions

Authors: Ivy Zhang, University of Minnesota – Twin Cities and Yong Zhang, Hong Kong University of Science & Technology. Working Paper: (http://ssrn.com/abstract=930725).

Summary: This study investigates acquirers' allocation of purchase price between goodwill and identifiable intangible assets upon the completion of acquisitions. The researchers compare the accounting for identifiable intangible assets pre- and post-FASB 142.

Study Construction:

If managers are concerned with intangible amortization which depress earnings, they will allocate more purchase price to goodwill in order to avoid future amortization expenses. Their first hypothesis is that research indicates that CEOs close to the end of their tenure focus more on short-term earnings rather than long-term performance to maximize their compensation.

They also investigated whether post-FASB 142 an acquirer with a higher market-to-book ratio prior to the acquisition allocates more purchase price to goodwill relative to identifiable intangible assets than expected. In addition, they also investigated whether post-FASB 142, an acquirer with multiple reporting units allocates more purchase price to goodwill relative to identifiable intangible assets than expected.

The sample selection is from all acquisitions completed between July 2001 and October 2005 reported by Securities Data Corporation. The study required that both the target and acquirer be publicly traded and that the target's primary SIC industry be business services. Various regression equations were created to measure the pre- and post impacts of FASB 142.

Findings:

The researchers found that firms with a high market-to-book ratio, unverifiable assets and multiple reporting units allocate more purchase price to goodwill. They also find that older CEOs allocate more purchase price to goodwill to reduce intangible amortization. An important finding for appraisers was that the researchers also found that external appraisers can constrain management's manipulation of the initial valuation of goodwill and other intangible assets but not eliminate it.

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Owner's Lack of Diversification and the Cost of Equity Capital for a Closely Held Firm

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Capital market theory suggests that undiversified investors may have a higher cost of capital than do diversified investors. Private companies generally trade at a discount to similar publicly traded companies. From a discounted cash flow viewpoint, this implies that the cost of capital for the private company is higher. We suggest that a large component of the private company discount is due to the entrepreneurial investor's lack of diversification. Using basic capital market theory, we provide a cost of capital model that depends on total risk faced by the entrepreneurial investor. We then propose a methodology based on a certainty equivalent approach and Monte Carlo simulation to quantify this risk. We also provide a simple case to illustrate its application. Our results suggest that the risk borne by the undiversified entrepreneurial investor is a major contributor to the private company discount through the cost of capital.

The valuation of privately held entrepreneurial firms differs from the valuation of publicly traded companies. Two defining differences are the illiquidity of the private firms and the fact that most private firms, especially the smaller ones, are owned by one or a few entrepreneurs with most of their wealth tied to their companies. There is an extensive and controversial body of research that attempts to quantify the illiquidity issue, also known as the discount for lack of marketability.[1] The second element, the entrepreneur's lack of diversification, is largely unexplored and this is the focus of the present paper. An implication of this article is that the private company discount is composed of at least two factors: the risk faced by the undiversified entrepreneur and illiquidity.

The commonly used income approach to valuation is based on discounted cash flows. This model discounts future cash flows at a rate appropriate for the expected risk of the expected income stream. The discount rate or the firm's cost of capital is also the investor or investors' minimum required return, and it depends on the risk free rate and a risk premium demanded by investors. The risk premium for the undiversified investor involves incorporating an appropriate compensation both for systematic risk (the risk that relates to market wide factors which affect all the securities) and for unsystematic risk (the risk that is firm specific). Typically, appraisers of private companies use an adjusted CAPM-based discount rate based on systematic risk premiums adjusted for industry and size. CAPM assumes that investors hold diversified portfolios and assesses the risk of a company based on its contribution to the risk of a diversified portfolio of assets. This is a reasonable assumption, and CAPM is a reasonable model for the cost of capital for publicly traded firms owned by a large group of investors, most of them minority, diversified investors. After using this cost of capital, the result is discounted for illiquidity to arrive at a private company value.[2] The next section presents the theoretical foundations for the lack of diversification discount on the top of the illiquidity discount

1. Cost of Capital Model for the Undiversified Investor

The cost of capital, or the entrepreneur's required return, for a private firm held by an undiversified entrepreneur who commits a significant fraction of his monetary wealth and human capital to the business, should reflect not only the systematic risk the entrepreneur faces, but also unsystematic risk, because it cannot be diversified away. Thus, when valuing entrepreneurial firms, the income-based appraisal should assess the lack-of-diversification risk premium to be added to the standard adjusted CAPM or build up model. Often this has been handled as a matter for the appraisers' subjective judgment, as reflected in discount to values obtained with the CAPM or other adjustments to the company's cost of capital, often called "firm-specific risk," usually with no support.

In this paper we suggest a methodology for incorporating the entrepreneur's loss-of diversification risk into a private company valuation and estimating the appropriate cost of capital.

1 It is not the purpose of this paper to discuss these issues. The reader is referred to Shannon Pratt's *Business Valuation Discounts and Premiums* (New York: John Wiley & Sons, 2001) for a detailed discussion of the issues.

2 There may be adjustments for control, but this is another, controversial, issue, which we are not addressing here.

The most popular set of models used to calculate the cost of equity capital of private companies are the ones based on CAPM. We ignore here the industry and size adjustment and we assume that they are incorporated in the market risk premium.

$$r_j = r_f + \beta_j (r_m - r_f) = r_f + \frac{\rho_{jm}\sigma_j}{\sigma_m}(r_m - r_f) \qquad (1)$$

where

r_j is the cost of equity for company j,

r_f is the risk free rate,

β_j is the beta of company j,

r_m is the market return ,

ρ_{jm} is the correlation between the returns on company j and the market,

σ_j is the standard deviation of the returns of company j, and

σ_m is the standard deviation of the market return.

Model (1) is based on the standard CAPM assumption that investors are diversified and they are concerned with non-diversifiable, systematic market risk as measured by beta. However, the entrepreneur is undiversified and bears the total risk of company j, which is more than the systematic risk of company j. Thus, the risk appropriate to the entrepreneur's investment should be based on total risk, as shown below.

To derive the cost of capital of the undiversified entrepreneur, we assume that the entrepreneur is indifferent between an investment in a diversified portfolio (i.e. the market portfolio), with the largest risk premium per unit of risk (highest return to risk ratio), and an undiversified investment in a private firm, which provides an equal return per unit of risk. We use the Sharpe Ratio as a benchmark because it is an ever present market-based alternative.[3] Using the Sharpe Ratio, we have:

$$\frac{r_m - r_f}{\sigma_m} = \frac{r_{ju} - r_f}{\sigma_j} \quad (2),$$

where,

r_{ju} is the required return by the entrepreneur to invest in the undiversified security, namely, the private firm.

Rearranging relation (2) we obtain:

$$r_{ju} = r_f + \frac{\sigma_j}{\sigma_m}(r_m - r_f) \quad (3)$$

If the return standard deviation of the private company is higher than the market's, then investors will require a higher return on that investment in order to keep their reward-to-risk ratio equal to that of the market.

Although the cost of capital model (3) has a solid theoretical foundation and is easy to understand, the difficulty lies in its practical application. The risk free rate, market return and standard deviation of the market can be estimated using public market data. However, the critical issue is estimating the standard deviation of private company returns. The CAPM is based on equilibrium price returns in the public markets, but returns based on share prices are unobservable for private companies. One commonly used way in practice to estimate the private company volatility is to assess the volatility of a set of comparable public companies. However, this approach though straightforward, has some important caveats. First, many private companies are small and may not be sufficiently similar to public companies for purposes of a volatility comparison. Thus, the use of larger and more established public companies, which are not exactly comparable, may understate significantly the volatility of the private companies' cash flows, and thus understate the risks facing the undiversified entrepreneurial investor. This is one of the reasons why, for example, venture capitalists use discount rates in the 30% to 50% range to discount projected new venture cash flows[4]. Because the entrepreneur is completely undiversified, identical cash flow claims are more risky for undiversified entrepreneurs than for the diversified investors in small public companies with similar cash flows.

Second, the entrepreneurial investor faces cash flow risks, which may not be represented well by stock price volatilities. Pubic companies' stock price volatilities may not represent the cash flow risks faced by the entrepreneurial investor. The most suitable comparable public companies in terms of firm characteristics often trade in the over-the-counter market. These companies may experience wild stock price fluctuations. Volatilities estimated from these companies' stock prices may be excessive as suggested long ago by Robert Shiller.[5] To the extent that they are excessive relative to the risks of the cash flows, they overstate the risks facing entrepreneurial investors and will not be a good proxy for estimating total risk. In the next section we suggest an alternative way to estimate the total risk faced by the undiversified entrepreneur.

2. Certainty Equivalent Approach

The primary goal of this article is to suggest a practical method to estimate the undiversified entrepreneur's cost of equity, by using a method that has the advantage of not

3 See: Sharpe, W. (1966), "Mutual Fund Performance," Journal of Business (January 1966), pp. 119-138

4 See: Jeffrey A. Timmons, *New Venture Creation*, 5th ed. (Chicago: Irwin, 1999), p. 465.

5 Robert Shiller (1981), "Do Stock Prices Move Too Much to be Justified by Subsequent Changes in Dividends?", *American Economic Review (71, 3), 421-36.*

relying on potentially excessive volatility estimates based on public companies' stock price data. As shown in model (1) above, the typical valuation approach is to discount the expected cash flows by a CAPM market based discount factor. The method presented in this paper is known as the Certainty Equivalent (CE) approach.[6] The CE approach adjusts the risky cash flows to their certainty equivalents and then discounts them at the risk free rate.

The certainty equivalent cash flow is defined as the smallest certain cash flow for which an investor will exchange a risky cash flow. The one period CE method has been discussed theoretically by Gordon Sick (1986) and others, but it has not been widely applied to valuing private companies. We present a practical approach to use the CE method to determine the cost of equity capital for a private company with an undiversified owner.

The risk-adjusted discount rate method and the certainty equivalent method lead in theory to the same present value, as model (4) shows.

$$PV = \frac{CF}{1+r} = \frac{CEC}{1+r_f} \quad (4)$$

where:

the Certainty Equivalent Cash Flow (CEC) = CF - $b(r_m - r_f)$,

r is the required return from CAPM , and

b is the *cash flow* beta.

Consistent with Model (3), for the undiversified investment in a private firm,

$$b = \sigma_{c/} \sigma_m ,$$

where:

σ_m is the market equity volatility, and

σ_c is the standard deviation of the private firm's cash flow (do not confuse with σ_j the standard deviation of the firms stock returns) .

The two methods differ in where the risk adjustment occurs. For the standard discounted cash flow method the risk adjustment is done in the denominator. The Certainty Equivalent method, on the other hand, discounts and risk adjusts the risky cash flows in the numerator.

Using the CE Approach, the undiversified entrepreneur would value the private company using:

$$PV = \frac{CF}{1+r} = \frac{CF - \frac{\sigma_c}{\sigma_m}(r_m - r_f)}{1+r_f} \quad (5)$$

Equation (5) shows that knowing the expected cash flow (CF), risk free rate, market volatility and the standard deviation of the firm's cash flows allows one to value the private company and calculate the implicit cost of equity capital that incorporates the lack of diversification of the owner.

The task of estimating the cash flows volatility is not as complex as it looks, since the appraiser has access to historical as well as forecasted cash flows. Using company-provided cash flow projections, estimates, which are standard and necessary for the valuation process, the volatility of the forecasted cash flows can be obtained using a simple scenario analysis or, as we will show below, using Monte-Carlo simulations of the cash flows.

The reader should note that models (4) and (5) calculate the present value of a single cash flow, compared to the standard discounted models that consist of a sum of multi-periodic discounted cash flows. We use a single period model for the following reasons. Theoretically, the Certainty-Equivalent infinite period model is elegant, but it is very cumbersome to use in practice. Furthermore, our focus in this paper is not on the calculation of the value of the firm but on estimating the one period cost of equity capital. Practical valuation models typically apply an average discount rate that is fixed across time periods. Thus, what we estimate is the multi-period representative discount rate of the undiversified investor in a private company.

We can then calculate the corresponding discount rate for an undiversified owner by rearranging model (4) as:

$$r_{ju} = \frac{CF}{CEC}(1+r_f) - 1 \quad (6)$$

where CEC is calculated using model (5), and the risky cash flow (CF), market volatility and the risk free rate were described earlier.

3. Case Study Illustration: Midas Manufacturing

The standard Income Approach values a company as the present value of future cash flows. The problem with the common approach when applied to a private company, is not the method, but the discount rate, which does not consider the fact that for most private companies the investor is undiversified. Here we provide an illustrative case study example that shows the reader how one can use the Certainty Equivalent method and Monte Carlo simulations to derive the cash flow beta and cost of equity capital for an undiversified owner.

Table 1 shows the income statement and cash flow projections for the next five years for Midas Manufacturing, a

6 Sick, Gordon, "A Certainty-Equivalent approach to capital budgeting" Financial Management 15: 4 23-35 1986; Kerins, Frank, Smith Janet and Richard Smith "Opportunity Cost of Capital for Venture Capital Investors and Entrepreneurs," Journal of Financial Quantitative Analysis 39.2, 2004; Grinblatt, Mark and Sheridan Titman, Financial Markets and Corporate Strategy, 2nd ed. 2002 McGraw-Hill. .

small, private, manufacturing company. The valuation date is the end of 2006, and, for simplicity, we assume that the company has no debt. The income statement provides a simplified cash flow calculation method since the focus of this case is on cost of capital and cash flow assumptions. The 2007 Sales are simple one year projections, with the subsequent annual sales growing at 5% a year. The COGS are projected to be 58% of Sales for 2007, 57% for 2008 and 2009, and 56% for 2010 and 2011. The SGA expenses are projected at 21% for 2007, then dropping to 20% for 2008 and 2009, and 19% for 2010 and 2011. Depreciation is assumed to be 9% of Sales. Taxes are 32% of EBIT. Capital expenditures are 1.1 times the despreciation, and changes in working capital as 8.6% of change in sales. The net cash flows are then discounted using the 16.42% cost of equity capital for a diversified investor. The cost of equity capital calculations are presented in Table 2, and follow a commonly used approach in valuation. Then, we calculate the average of the net cash flows over the five years, average we consider representative of the expected one period cash flow.

After the expected one period cash flow is calculated, the appraiser needs to conduct a Monte Carlo simulation of the cash flow model (i.e., Table 1) in order to estimate the cash flow volatility. Table 3 presents the assumptions of Monte Carlo simulation, the results of the simulation (i.e., standard deviation of cash flow) and the calculations of the Certainty Equivalent Cost of Equity Capital.

To apply the CE valuation formula (6) we need to estimate first the volatility of the cash flows. This can be accomplished using the Monte Carlo simulations of the future cash flows. Many of the world's largest financial institutions use Monte Carlo risk analysis for capital budgeting, valuation and investment decisions. Monte Carlo analysis is a simulation method which randomly and repeatedly generates values for uncertain variables in order to simulate many possible outcomes, in our case, future cash flows. We extend the spreadsheet (table 1) by using CrystalBall® to determine the most probable value for the Mean Cash Flow over 2007-2001 (the one period expected cash flow CF in the models) and its distribution. The inputs modeled in the simulations are: (i) growth rate of Sales, with a mean of 5% and standard deviation under normal distribution of 2.5%; (ii) COGS with a triangular distribution with an expected percentage of Sales of 58%, minimum of 54% and maximum of 62%; (ii) SGA with a triangular distribution with an expected percentage of Sales of 21%, minimum of 16% and maximum of 26%.

The Monte Carlo simulation was conducted by running 10,000 iterations. The simulation provides the volatility of the Mean Cash Flow over 2007-2001, the critical input of Certainty Equivalent model, of $570,723. We need to point out that the σ_c in (5) is measured in dollars. It should be remembered that σ_m is measured in percentage returns, such that b times the market risk premium has the same unit of measure as CF.

Next, we use the Certainty Equivalent Cash Flow CEC (from model (5)), and apply model (5) to calculate the present value based on a one period model and the capital markets inputs (i) risk free rate equal to 4.92% (same one used for the build-up model), (ii) expected market risk premium of 7.1% from SBBI (matches the number from the build-up model in Table 2); (iii) volatility of the market return of 20%, SBBI 2006 . The results are reported in table 3. Using Model (5) we find that the Certainty Equivalent Cash Flow (CEC) is equal to (CF - b * Market Risk Premium), which equals $1,324,128.

Using Model (6), with the CEC calculated above, we find the Cost of Equity Capital for an undiversified investor in a private company is:

$$r_{ju} = \frac{\$1,526,735}{\$1,324,128}(1 + 0.0492) - 1 = 0.2097 \text{ or } 20.97\% \quad (9)$$

The "undiversified" discount rate of 20.97% is almost 4.5 percent higher than the discount rate of 16.42%, the discount rate calculated using the build-up model and assuming a diversified ownership.

Table 4 provides the cost of equity capital for the undiversified and the diversified investor, as well as the firm values calculated using the simple constant growth model.

Using the inputs from discounted cash flow analysis in Table 1, namely the mean cash flow of $1,526,735, a growth rate of cash flow of 5%, risk-free rate of 4.92%, and the respective cost of equity we find a firm value of $13,366,618 when using the cost of equity capital for the diversified investor (16.42%), compared to a firm value of $9,557,655 when using the cost of equity capital for the undiversified investor (20.97%)

These results suggest a "private company" discount for lack of diversification of 28.50% (1 - $9,557,655 / $13,366,618).

4. Relation to Private Company Discounts and Illiquidity

The discount we calculate above accounts for the lack of diversification facing the entrepreneur. Pure illiquidity is another issue. Bajaj et al.[7] may have observed this factor. They find it to be approximately 7%. However, it is a mistake to consider the discount observed by these authors to represent the entire discount for a private company relative to a

7 M. Bajaj, D. Denis, S. Ferris, and A. Sarin, "Firm Value and Marketability Discounts," Journal of Corporation Law, Fall 2001, 89-115.

public company. In fact, Koeplin, Sarin and Shapiro[8] have empirically determined that the private company discount is on the order of 20% to 30%. This empirical discount would incorporate lack of diversification and illiquidity. The private company lack of diversification discount that we calculate, combined with those of Bajaj et al. is in the range suggested by Koeplin et al. and is in a range that is found in practice:

$$1 - [(1 - 0.285) \text{ X } (1 - 0.07)] = 33.5\%.$$

The results of our model suggest that the private company discount is composed of two major factors: the risk faced by an undiversified investor and illiquidity.

5. Conclusion

We have shown, using basic capital market theory, that an undiversified investor will require a higher rate of return determined by his or her exposure to risk than would a diversified investor. The benchmark for return to risk is obtained from the Sharpe ratio. We utilize a certainty equivalent model to discount the risky cash flows, and we give an illustrative application using Monte Carlo simulation to derive the risk of cash flows. Our model suggests that much of the private company discount is due to a lack of diversification. We have not dealt with the issue of illiquidity per se, but this is a fruitful area for future research when combined with the notion that capital asset pricing theory suggests that the private company discount is partially a discount due to the higher returns required by undiversified investors.

Another benefit of using our model is that it does not use a risk measure based on the stock price volatility of market comparable companies. This avoids using possibly problematic volatility measurements due to excess volatility or inappropriate market comparables. It also utilizes a risk measure based on cash flow risk, which is the primary risk faced by the entrepreneurial investor.

One might argue that this model results in an investment value specific to the buyer and not fair market value. We do not agree with this because the idea of fair market value assumes a typical buyer. For smaller companies, the typical buyer will be the single entrepreneur. For larger private companies, the typical buyers may be private equity funds or large corporations that are partially diversified, and the prices will be higher because of lower required returns.

McConaughy, Daniel L., Covrig, Vincent.
"Owner's Lack of Diversification and the Cost of Equity Capital for a Closely Held Firm."

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8 J. Koeplin, A. Sarin, and A. Shaprio, "The Private Company Discount," *Journal of Applied Corporate Finance* 12.4, 94-101.

Table 1: Midas Manufacturing
Projected Cash Flows

	2006	2007	2008	2009	2010	2011
Sales		$16,488,292	$17,312,707	$18,178,342	$19,087,259	$20,041,622
Growth, %			5.0%	5.0%	5.0%	5.0%
COGS		9,563,209	9,940,956	10,333,624	10,741,802	11,166,103
COGS as a % of Sales		58.0%	57%	57%	56%	56%
Gross Profit		6,925,083	7,371,750	7,844,718	8,345,457	8,875,519
SG&A Expenses		3,462,541	3,544,777	3,628,965	3,715,153	3,803,388
SG&A as a % of Sales		21.0%	20%	20%	19%	19%
Depreciation		1,483,946	1,558,144	1,636,051	1,717,853	1,803,746
Depreciation as a % of Sales		9.0%	9%	9%	9%	9%
EBIT		1,978,595	2,268,830	2,579,702	2,912,451	3,268,385
Less: Taxes @ 32% of EBIT	32.0%	633,150	726,026	825,505	931,984	1,045,883
Net Operating Income		1,345,445	1,542,805	1,754,197	1,980,466	2,222,502
Plus: Depreciation		1,483,946	1,558,144	1,636,051	1,717,853	1,803,746
Less: Capital Expenditures	1.10	1,632,341	1,713,958	1,799,656	1,889,639	1,984,121
Capital Expenditures as a % of Sales		9.9%	9.9%	9.9%	9.9%	9.9%
Less: Change in Working Capital	8.60%	70,900	74,445	78,167	82,075	86,179
Adjusted Net Cash Flow		$1,126,150	$1,312,546	$1,512,426	$1,726,606	$1,955,948
Mean Cash Flow over 2007-2011	$1,526,735					
Volatility of Cash Flow based on Monte Carlo Simulation	570,723					

Table 2: Midas Manufacturing

Cost of Equity Capital (for a diversified investor)

Build-up Model	
Risk Free Rate (1)	4.91%
Beta (2)	0.72
Equity Risk Premium (3)	7.10%
Micro-Cap Risk Premium (4)	6.40%
Cost of Equity Capital	**16.42%**

(1) Represents December 31, 2006 20 year treasury rate per www.federalreserve.gov/releases/H15/data/wf/tcm20y.txt.

(2) For illustrative purposes only

(3) Long-horizon expected risk premium S&P500 index SBBI Valuation Edition 2006

(4) Size Premium, Group 10, last page SBBI Valuation Editions 2006

Table 3: Midas Manufacturing
Entrepeneur's Equity Cost of Capital
Certainty Equivalent Method

1. Monte Carlo Simulation Assumptions

Growth Rate of Sales follows the Normal Distribution with:		
Expected Growth rate of Cash Flow	5%	
Standard Deviation of the Growth Rate of Cash Flow	3%	
COGS follows a triangular distribution with:		
COGS as a % of Sales	58%	
Min	54%	
Max	62%	
SGA Expense follows a triangular distribution with:		
SGA as a % of Sales	21%	
Min	16%	
Max	25%	

2. Monte Carlo Simulation Results

Mean Cash Flow 1	$1,526,735	
Standard Deviation of the Cash Flow	$570,723	

3. Certainty Equivalent Method

Risk free rate	4.92%	
Market Volatility	20%	SBBI 2006, annual returns 1926-2005
Cash Flow Beta	$2,853,615	
Market Risk Premium	7.10%	SBBI, matches the COEC
Certainty Equivalent Cash Flow	$1,324,128	
Present Value of CF based on the Certainty Equivalent Method	$1,262,036	
Cost of Equity Capital (Certainty Equivalent Method)	**20.97%**	
Cost of Equity Capital (Build-up Method)	**16.42%**	

Table 4. Firm Values and the Lack of Diversification Discount

Cash Flow at time 1	$1,526,735
Growth rate of Cash Flow	5.00%
Risk Free Rate	4.92%
1. Firm value calculations using the Constant Growth model and Cost of Capital for the diversified investor(Build-up Method)	
Cost of Equity Capital (build-up model)	16.42%
Firm Value (constant growth model)	$13,366,618
2. Firm value calculations using the Constant Growth model and Cost of Capital for the undiversified investor (Certainty Equivalent)	
Entrepeneur's discount rate (Certainty Equivalent Method)	20.97%
Firm Value (constant growth model)	$9,557,655
Lack of Diversification Discount	**28.50%**